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A modified double –torsion method for measuring the fracture toughness of polymeric ophthalmic lenses

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A modified double torsion test has been developed to measure the fracture toughness of polymeric ophthalmic lenses. Measurements on PMMA sheet using both conventional double torsion and compact tension geometries gave the same fracture toughnesses as lenses made from the PMMA sheet and tested in the modified double torsion test. Tests on plano lenses made of commercial ophthalmic lens materials showed that the measured fracture toughness was independent of lens diameter and thickness and also crack length provided the notch length was within specified bounds. Test results showed quite small scatter. Because of the reproducibility and simplicity of the test, the modified double torsion test should provide a more reliable and cheaper measure of the impact performance of the lenses than does the existing drop weight test.

1. Introduction

Ophthalmic lenses are lenses intended for use in spectacles. The ones used in this research consisted of a thin-walled, shallow, spherical shell made of rigid polymer. For simplicity, the test lenses were plano, meaning that they had parallel surfaces, and so were optically neutral.

Ophthalmic lenses require a number of properties such as scratch resistance, rigidity and impact resistance in addition to their optical characteristics. Unfortunately, improvements to some of these properties, such as scratch resistance and rigidity, can lead to a decrease in the impact resistance of the material. Therefore, when developing new lens materials it is important to have an accurate and inexpensive method of measuring the impact resistance.

The current method of evaluating the impact resistance of lenses is to drop weights onto the lenses from varying heights. It is then possible to determine the impact energy required to cause fracture. This is similar to the method described in ASTM D3029-90 for measuring the impact resistance of flat polymer sheets [1]. This method is expensive in time and materials and the results are subject to variation due to the natural distribution of flaws in the lenses. These factors make this test unsuitable for use in the development of new materials.

It would be preferable to directly measure the material property of fracture toughness. Two measures can be made of fracture toughness. One measure is the work required to grow a crack by unit area, the fracture resistance R. At fracture the elastic energy released by crack growth (G) has to be equal or greater than R. The second way is the critical stress intensity factor, K_{1c}. This is a measure of the stress intensity factor at a crack tip when crack growth occurs. The stress intensity factor, K_1, is related to the applied stress, σ, and the crack length, a, by

$$K_1 = Y\sigma\sqrt{(\pi a)} \quad (1)$$

where Y is a geometry factor. At fracture the critical stress intensity factor K_{1c}, is related to G_{1c} by

$$K_{1c} = \sqrt{(G_{1c}E^*)} \quad (2)$$

where $E^* = E$ (the elastic modulus) when the sample is in plane stress, and $E^* = E/(1 - \nu^2)$ for plane strain. ν is Poisson's ratio.

Standard methods [2] for measuring K_{1c}, such as compact tension and notched three-point bend tests, cannot be used for lenses as lens specimens are in the shape of thin-walled, shallow, spherical shells rather than a flat sheet or bar. Thus the lenses require the use of a new measurement technique. This paper presents the results of an evaluation of a different measurement method for dealing with this problem.

There are a number of ways in which established test methods for fracture toughness might be modified to account for this geometry. Plati and Williams [3]. and Brown [4] have used a pendulum impact tester to measure the fracture toughness of thermoplastics. They measured the impact energy (U) required to break a notched sample, and this is related to G_{1c} by:

$$U - U_k = G_{1c}BD\Phi \quad (3)$$

where U_k is the losses due to friction and the kinetic energy imparted to the fragments, B is the breadth of the sample, and D is the depth of the sample. Φ is a function of crack length and D. By plotting U against $BD\Phi$ one obtains a line with a slope equal to G_{1c}. Since the radius of curvature of lenses is generally large, it might seem reasonable to ignore it and use a pendulum impact test. However, when strips 10 mm wide were cut from the centre of the lenses, notched to various depths, and tested in a pendulum impact machine, the results had a level of scatter which was too large for the result to be useful. The large scatter resulted from the fact that the lens materials were generally quite brittle and the energy needed to break the sample was of the same magnitude as the energy loss term U_k. It is under these conditions that scatter due to the vibration between the specimen and the impact hammer can be severe [5].

Another testing method used to measure K_{Ic} of brittle materials is the indentation crack technique, in which a Vickers pyramid indenter is pressed into the material under a set load until radial cracks are formed from the corners of the indentation. The fracture toughness is calculated from the load and the crack length [6, 7]. This method was abandoned, because the penetration of the indenter into the polymer was so large that the support of the indenter fouled the test specimen before well-formed cracks could form.

Another approach to testing specimens with the lens geometry would be to test, in compression, lenses precracked in the centre of the concave surface. A semicircular notch could be produced in the centre of the concave side of the lens and the lens fractured by applying a point load at the centre of the convex side of the lens. Using a numerical analysis of the stress in a shallow spherical shell [8] combined with the formula for a flat plate containing a semicircular surface notch subject to a bending moment [9], the fracture toughness can be shown to be

$$K_{Ic} = 6B_1P/4\pi\sqrt{(\pi a/t^2)} \quad (4)$$

where P is the applied load, a is the crack depth, t is the thickness, and B_1 is a geometry factor. While more useable than the pendulum impact tests, this method also proved too inaccurate to be of use. The large scatter obtained for this method was probably a result of variations in the notch sharpness since the shell geometry of the lenses made it difficult to produce consistent notches. To circumvent this problem, the test geometry was changed to one of essentially double torsion.

2. Modified double torsion

The method of double torsion [5, 6, 10] has been used previously to determine the fracture toughness of crosslinked polymers [11,12], but it has not been tried with curved samples. Standard double torsion specimens consist of a flat rectangular plate with a sharpened crack running part way down the longitudinal axis, Fig.1. This gives two ,parallel cantilever arms which are joined at the crack tip. Deflection controlled loads are applied at four points to produce opposing torsion in the cantilever arms. This has been described by Outwater *et al.* [13] and by Kies and Clark [14]. The mode of failure for four-point loading was analysed by Evans [10] and shown to be Mode I.

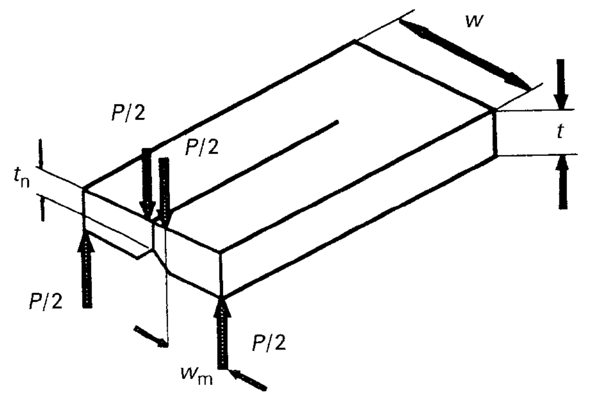

Figure 1 A sample for ordinary double torsion showing the dimensions and applied loads.

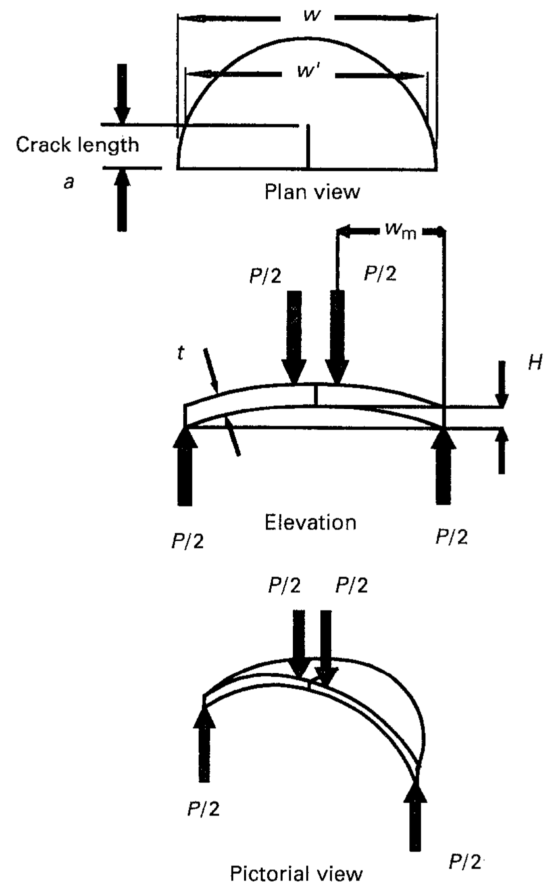

Figure 2 The modified double torsion sample showing applied loads ($P/2$).

At a critical amount of torsion, the crack begins to grow, and the stress intensity factor is given by the formula [6]:

$$K_{Ic} = PW_m\sqrt{(3(1+\nu)/(Wt^3t_n))} \quad (5)$$

where P is the load as shown in Fig. 1, W_m is the length of the moment arm, ν is Poisson's ratio, W is the width of the sample at the crack tip, t is the thickness of the sample, and t_n is the thickness of the sample along the crack path.

By cutting a lens in half and introducing a sharpened notch (pre-crack) into the flat edge, a modified double torsion sample was made, Fig. 2. These samples were then loaded in double torsion as shown.

Assuming that the crack opening is still Mode I (an issue that will be discussed further in Section 4.3), Equation 5 must now be modified to allow for the different geometry of the lenses. In this analysis, the stress state in the lens centre was assumed to be that of a flat plate in double torsion. Vlasov and Leont'ev [15] have shown that provided the H/W ratio for a dome is less than $\frac{1}{8}$, it behaves under strain like a flat plate. Lenses tested here all had H/W ratios below $\frac{1}{10}$ and consequently, it was considered that no corrections were required to take into account the curvature of the lenses.

The strain energy release rate, G_{1c}, for crack growth is given by the formula

$$G_{1c} = (1/2)P^2 \mathrm{d}C/\mathrm{d}A \qquad (6)$$

where $\mathrm{d}C$ is the change in compliance (the deflection per unit applied force), $\mathrm{d}A$ is the change in the surface area of the crack, and P is the applied load. As the thickness t_n is constant, the increase in crack area is:

$$\mathrm{d}A = t_n \mathrm{d}a \qquad (7)$$

where $\mathrm{d}a$ is the increase in crack length.

The double torsion specimen can be pictured as two parallel sided cantilever beams in torsion. These beams are subject to a twisting moment of $PW_m/4$. The compliance of such a beam is given by the equation [5]

$$C = 3(1 + \nu)W_m^2 a/(Et^3 W) \qquad (8)$$

For a semicircular sample, the width is now a function of crack depth and is given by $W' = 2\sqrt{(r^2 - a^2)}$. Substituting this into the formula for the compliance, it is found that for each increment of crack growth $\mathrm{d}a$,

$$\mathrm{d}C = 3(1 + \nu)W_m \mathrm{d}a/(Et^3\sqrt{(r^2 - a^2)}) \qquad (9)$$

The total compliance of a sample with crack length a is found by integration of equation (9) to be

$$C = 3(1 + \nu)W_m/(Et^3)\sin^{-1}(a/r) \qquad (10)$$

Therefore

$$\mathrm{d}C/\mathrm{d}a = 3(1 + \nu)W_m^2/(Et^3\sqrt{(r^2 - a^2)}) \qquad (11)$$

In addition it was found that side grooves on the lens specimen were not required, so that t_n was equal to t. Combining Equations 2, 6 and 11

$$K_{1c} = PW_m\sqrt{(3(1 + \nu)/(\sqrt{(r^2 - a^2)}t^4))} \qquad (12)$$

which has the same form as Equation 5 for a conventional, parallel-sided sample.

3. Experimental method

3.1. Materials

Most of the resins used in this study were thermosetting resins supplied by Sola International Holdings Ltd. The resins were designated R1, R2 and R3. R1 was diallyl diethylene glycol carbonate while R2 was a crosslinked acrylic material. R3 was a proprietary formulation especially developed to have superior toughness.

Most of the tests were performed on R1 and R2 lenses, which had a 167 mm radius of curvature, a diameter of 65 mm and a thickness of 3 mm. Further tests were done using R1, R2 and R3 lenses that were 75 mm in diameter and 1.8 mm thick.

As a check on the modified double torsion test method, a series of tests was also conducted on PMMA sheet, and on lenses fabricated from the sheet. The PMMA was obtained in the form of commercially available 2.8 mm thick sheet.

3.2. Construction of the test pieces

Five lenses were made from PMMA sheet by cutting out disks with a hacksaw, heating them over a hotplate until they were soft, and pressing them into shape between two of the R2 resin lenses. Flat double torsion and compact tension samples were also cut from the same sheet and subjected to the same thermal history.

To make the lenses into test pieces, a lens was cut in half using a fine toothed hacksaw. A precrack was cut into the sample with a thin coping saw as shown in Fig. 2. The cracks used were generally between 8 mm and 15 mm long, although one series of tests was conducted using a wider range of crack lengths to verify that the measured K_1 at fracture was independent of crack length. In order to get consistent results it was important to ensure that the initial precracks were very sharp. To achieve this sharpness the initial notch cut by a coping saw was sharpened by pressing a scalpel blade into the bottom of the notch so that a crack ran out ahead of the blade. This is a technique used with many brittle polymers to sharpen crack tips [5].

This method worked well for the R1 and R2 resins, but not for the R3 and PMMA samples. These latter resins were too tough for a crack to grow ahead of a scalpel pressed into the tip of a saw-notch. When a crack did appear it was unstable and the specimen usually failed during the precracking. Samples of R3 and PMMA that had the notch sharpened by a scalpel cut, rather than a sharp precrack, recorded about twice the toughness values of the properly cracked samples.

In order to get the R3 and PMMA samples properly precracked, it was necessary to hold the scalpel blade against the bottom of the notch, and then to hit the blade against the notch tip. This was done by lifting the scalpel and the sample up and tapping them sharply on a hard surface. Alternatively, the samples were held in place and the scalpel was struck on the back of the blade. Both of these methods caused the sudden appearance of a crack ahead of the scalpel blade. When the force of the blow was judged correctly, the crack was of the desired length and the sample was ready for testing. All initial cracklengths were measured after testing using an optical microscope.

3.3. Testing procedure

Each of the samples were placed in a jig that steadily increased the torsional deflection until the crack extended across the lens and the sample broke. This was done in an Instron 1026 tensile testing machine at a crosshead speed of 10 mm min^{-1}. All tests were conducted at approximately 25 °C.

To determine the accuracy of the method, a number of different specimens of each resin were tested. Initially, tests were performed on 15 samples of R2 to determine the repeatability of the results. Then 20 samples each of R1 and R3 were tested to enable the toughness values to be compared with that of R2.

Fracture toughness is a material property and consequently not affected by the specimen geometry. Therefore, specimen geometry effects were evaluated by a series of tests conducted on an additional two batches of R1 resin lenses that were 75 mm wide and 1.8 mm thick. The effect of notch depth was evaluated by testing the 65 mm diameter R1 resin lenses that were precracked to various depths between 2 mm and 20 mm.

To ensure that the derivation of the K_{Ic} formula was correct, the relationship between the compliance and the crack length was determined. The compliance of R1 resin lenses was determined from the slope of the load versus deflection curves for 25 modified double torsion samples with different values of a.

As a final check on the modified double torsion test and to compare the results of the test with those of known Mode I crack opening, experiments were done using PMMA samples. Lenses made from 2.8 mm PMMA sheet with a 65 mm diameter and a 167 mm radius of curvature were tested in the modified double torsion test. Conventional flat double torsion tests and compact tension tests were then both conducted on 50 mm by 60 mm by 2.8 mm samples of the same material, with the cracks running down the long axis of the sample. All these tests were done at a crosshead speed of 10 mm min^{-1}.

4. Results and discussion

The load versus deflection curve for samples tested in the modified double torsion tests increased linearly until the point of failure, Fig. 3. At failure, a crack grew rapidly across the sample. In the classical double torsion test, the crack grows at a steady load without sudden failure because K_I decreases as crack growth increases the compliance, and hence decreases the load. In the modified test, the value of W' decreased with increasing crack length, so as crack length increased, the critical load for crack growth decreased. Therefore the crack was unstable.

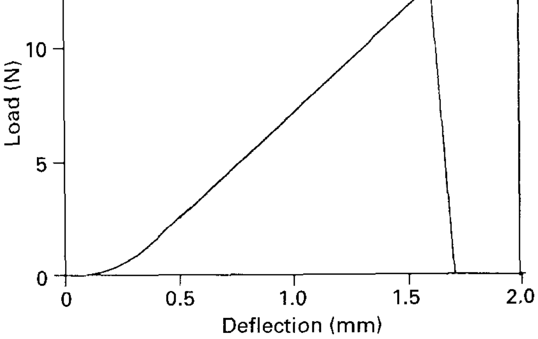

Figure 3 A typical load versus deflection curve for the modified double torsion test.

4.1. Compliance testing

The compliance of the 65 mm diameter, 3 mm thick samples calculated from the load deflection curves is shown as a function of crack length in Fig. 4. Also plotted in Fig. 4 is the theoretical compliance for specimens of this geometry calculated with Equation 10. An accurate value of the Poisson's ratio of the material was not available, so ν was estimated as being 0.3. Young's modulus was approximately 2 GPa. Those values gave good correlation between the measured compliance and the theoretical value except at high values of $a > 20$ mm. The deviation from the theoretical values of compliance at high values of a is because the analysis of a double torsion sample assumes that all deflection is due to torsion of the arms [10]. As the crack is approaching the back surface of the lens, the deflection due to the bending of the region ahead of the crack tip becomes significant, giving an increased compliance.

4.2. Comparison of resins

A series of 15 tests were done on each of R1 resin and R2 resin, using the 65 mm diameter, 3 mm thick lenses. The average K_{Ic} value for the R2 resin samples was 433 kPa$\sqrt{m}$ with a standard deviation of 23 kPa$\sqrt{m}$ (5% of the mean). This resin was slightly tougher than the R1 resin which gave a fracture toughness of 360 kPa$\sqrt{m}$ with a standard deviation of 28 kPa$\sqrt{m}$ (8% of the mean). These results are shown in Table I.

Tests done on R3 resin; 75 mm diameter, 1.8 mm thick lenses gave a toughness of 1075 kPa$\sqrt{m}$ with a standard deviation of 43 kPa$\sqrt{m}$, which was between two and a half and three times the values obtained for the other resins.

The most pleasing aspect of these results was the low level of scatter, with the standard deviations being of the order of 5% of the mean. This will allow quite

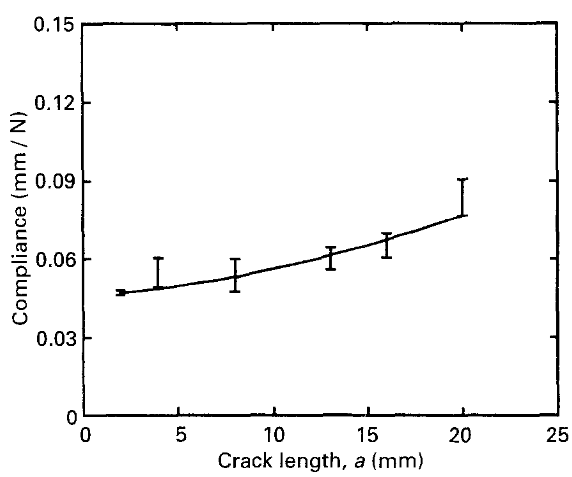

Figure 4 Compliance of R1 samples with a width of 65 mm, 3 mm thick, with different crack lengths. The solid line is the theoretical compliance curve given by Equation 10. Error bars show the standard deviation of measured results.

TABLE I Experimental results for the different resins showing the measured K_{1c} and the standard deviations

Resin and dimensions (mm)	Test method	Average K_{1c} (kPa$\sqrt{m}$)	Standard deviation (kPa$\sqrt{m}$)
R1 3 × 65	Modified D. T.	359	28
R1 1.8 × 75	Modified D. T.	356	38
R1 1.8 × 75	Modified D. T.	365	28
R2 3 × 65	Modified D. T.	423	20
R3 1.8 × 75	Modified D. T.	1075	43
PMMA 2.8 × 50 × 60	Compact Tension	1130	115
PMMA 2.8 × 50 × 60	Double Torsion	1173	55
PMMA 2.8 × 65	Modified D. T.	1214	75

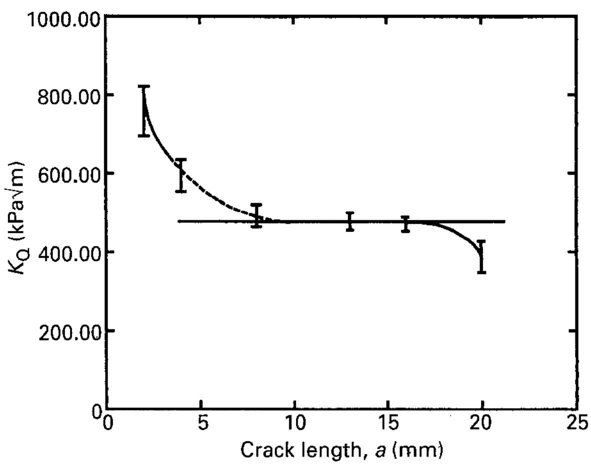

Figure 5 K_Q values versus crack length for R2 resin samples, diameter 65 mm, thickness 3 mm.

small changes in fracture toughness to be detected with a small number of samples. The confidence interval for a value (μ) is given by

$$(t - c\sigma/\sqrt{n}) \leqslant \mu \leqslant (t + c\sigma/\sqrt{n}) \quad (13)$$

where t is the average of the experimental results, c is a constant determined by the degree of confidence required, σ is the standard deviation and n is the number of results [16]. Hence, to get a given level of accuracy the method with a smaller value of σ will require a lower value of n. For example, to obtain a 95% confidence interval that is within 5% of the mean requires just 10 samples for the R1 resin. Therefore the modified double torsion test will allow an accurate measure of the fracture toughness to be obtained from fewer test samples than the 25–40 used in the drop weight test.

It was also pleasing to note that the ranking of the three resins corresponded with general experience of the resins in service.

4.3. PMMA lenses

The series of PMMA lenses tested using the modified double torsion method gave an average toughness value of 1214 kPa$\sqrt{m}$, with a standard deviation of 75 kPa$\sqrt{m}$. When flat PMMA plates were tested in double torsion, K_{1c} was found to be 1173 kPa$\sqrt{m}$, with a standard deviation of 55 kPa$\sqrt{m}$. Finally the same sheet of PMMA was tested using compact tension tests. This gave an average fracture toughness value of 1130 kPa$\sqrt{m}$, with a standard deviation of 115 kPa$\sqrt{m}$. These results are also shown in Table 1.

The value obtained for the PMMA lenses using the modified torsion test was marginally higher than those obtained for the flat samples but within one standard deviation of those latter results. This suggests that the assumption made about the negligible effect of lens curvature was valid. Moreover, the compact tension geometry is known to give Mode I crack opening and the conventional double torsion geometry is also accepted to give a valid K_{1c} [10, 13]. It might be expected that for relatively brittle materials like those tested here, mixed mode crack opening would result in some variation in the measured fracture toughness from that measured in Mode I opening. The fact that the same value of fracture toughness was obtained for PMMA for the three different test geometries used here suggested that the crack opening in the modified double torsion method remained Mode I and that the measured quantity was indeed K_{1c}.

4.4. Different sample geometry

A series of tests was conducted on two batches of R1 lenses that were 75 mm wide and 1.8 mm thick. The average fracture toughness values for the two batches of material were 356 kPa$\sqrt{m}$ and 365 kPa$\sqrt{m}$ which were within the standard deviation of those for the thicker and smaller diameter samples.

Fig. 5 shows the relationship of measured K_1 at fracture, K_Q, to crack length. Here it can be seen that K_Q had a dependence on a at very short crack lengths and again at very large crack lengths. The dependence of K_Q on a at very short crack lengths was most likely due to the fact that at small a, the sample was acting as though it were in a four-point bend test, not double torsion. At very large crack lengths the change in K_Q follows the change in compliance, Fig. 4, discussed in Section 4.1. However when the crack length is between 8 mm and 15 mm the measured K_Q result was independent of the crack length. This is the expected result for double torsion and further confirmed that the experiment gave a K_{1c} result.

5. Conclusion

A modified double torsion test has been shown to give accurate and consistent measures of the fracture toughness of ophthalmic lens materials. Provided the initial notch length was limited to between 8 mm and 15 mm for 65 mm diameter lenses, the fracture toughness values obtained were independent of specimen geometry.

Since the specimens were easy to prepare and the test method simple, the test method should prove a faster, cheaper and more reliable indicator of lens performance than the existing drop weight tests.

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